Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Gold Level (Harder) G2

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
63	**54**	**45**	**36**	**29**	**22**

1. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \qquad x \neq \pm 2$$

find the values of the constants a, b, c, d and e.

(4)

June 2013

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2. Given that

$$f(x) = \ln x, \qquad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x-axis.
In each case, state the equation of the asymptote.

(7)

Juen 2013

3.

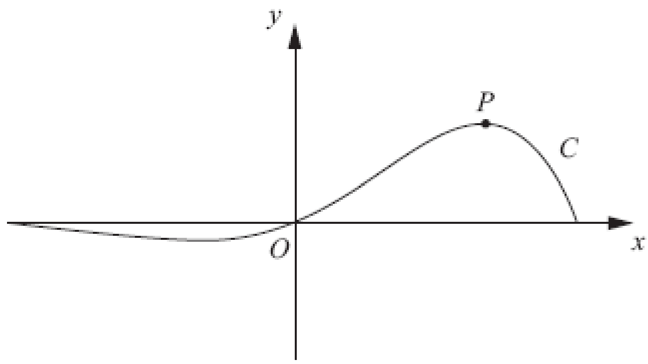

Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x, \quad -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}.$$

(*a*) Find the x-coordinate of the turning point P on C, for which $x > 0$.
Give your answer as a multiple of π.

(6)

(*b*) Find an equation of the normal to C at the point where $x = 0$.

(3)

June 2012

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4. The point P is the point on the curve $x = 2\tan\left(y + \frac{\pi}{12}\right)$ with y-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

January 2012

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5. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ °C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90 °C,

(*a*) find the value of A.

(2)

The tea takes 5 minutes to decrease in temperature from 90 °C to 55 °C.

(*b*) Show that $k = \frac{1}{5} \ln 2$.

(3)

(*c*) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places.

(3)

January 2011

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6. Find algebraically the exact solutions to the equations

(*a*) $\ln (4 - 2x) + \ln (9 - 3x) = 2 \ln (x + 1), \quad -1 < x < 2,$

(5)

(*b*) $2^x e^{3x+1} = 10.$

Give your answer to (*b*) in the form $\frac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers.

(5)

June 2013

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7. (*a*) Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\ \text{cosec}\ 2\theta, \qquad \theta \neq 90n°.$$

(4)

(*b*) Sketch the graph of $y = 2\ \text{cosec}\ 2\theta$ for $0° < \theta < 360°$.

(2)

(*c*) Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 3$$

giving your answers to 1 decimal place.

(6)

June 2007

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8. Solve

$$\text{cosec}^2\ 2x - \cot 2x = 1$$

for $0 \leq x \leq 180°$.

(7)

January 2010

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9. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = \mathrm{f}(x)$ is shown in Figure 1.

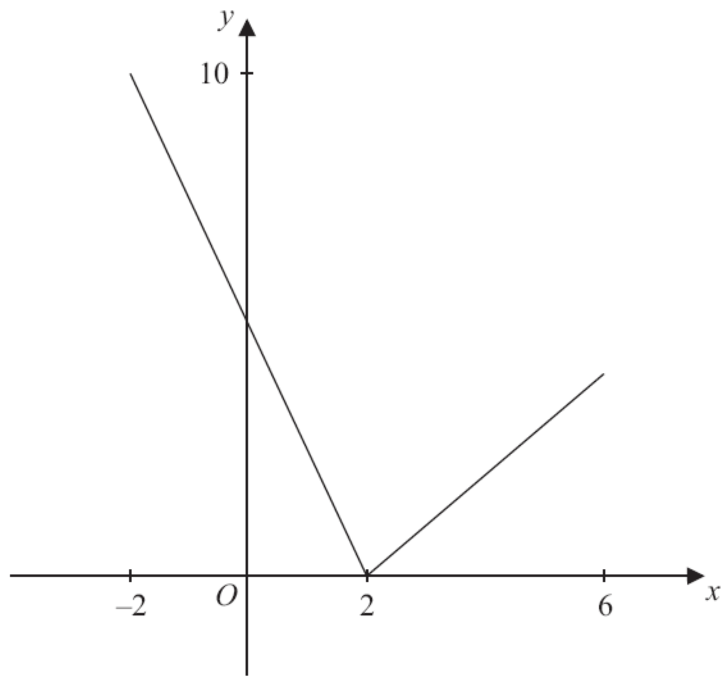


Figure 1

(*a*) Write down the range of f.

(1)

(*b*) Find ff(0).

(2)

The function g is defined by

$$\mathrm{g} : x \rightarrow \frac{4+3x}{5-x}, \qquad x \in \mathbb{R}, \qquad x \neq 5.$$

(*c*) Find $\mathrm{g}^{-1}(x)$.

(3)

(*d*) Solve the equation $\mathrm{gf}(x) = 16$.

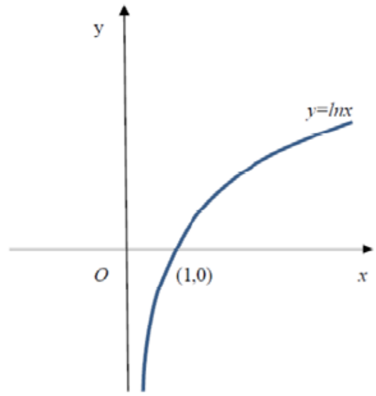
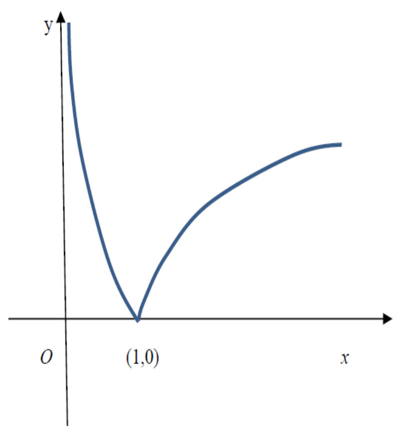
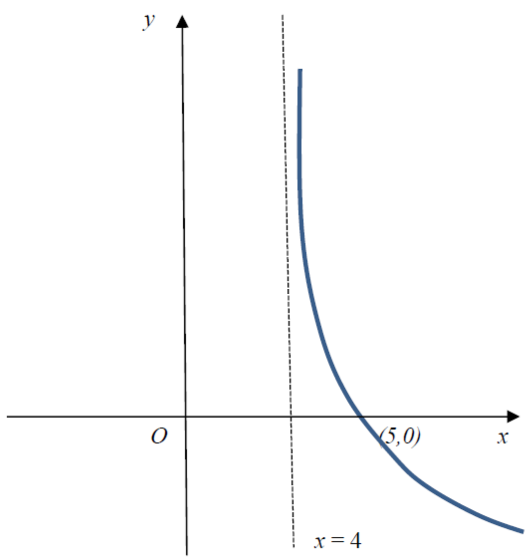
(5)

June 2013

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1 By Division	$$\begin{array}{r} 3x^2-2x+7 \\ x^2(+0x)-4\,\overline{\smash{)}\,3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+7x^2+0x \\ \underline{-2x^3+0x^2+8x} \\ 7x^2-8x-4 \\ \underline{7x^2+0x-28} \\ -8x+24 \end{array}$$	
	$a=3$	**B1**
	Long division as far as $$\begin{array}{r} 3x^2-2x....... \\ x^2(+0x)-4\,\overline{\smash{)}\,3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+................ \\ \underline{-2x^3+.................} \end{array}$$	**M1**
	Two of $b=-2 \quad c=7 \quad d=-8 \quad e=24$	**A1**
	All four of $b=-2 \quad c=7 \quad d=-8 \quad e=24$	**A1**
		[4]

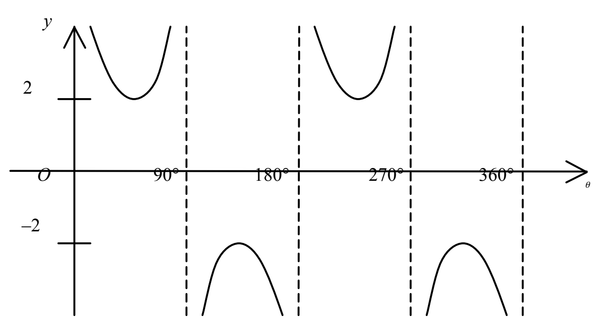
Question Number	Scheme	Marks
2(i)	 *ln* graph crossing x axis at (1,0) and asymptote at x=0	B1
2(ii)	 Shape including cusp Touches or crosses the x axis at (1,0) Asymptote given as x=0	B1ft B1ft B1
2(iii)	 Shape Crosses at (5, 0) Asymptote given as x=4	B1 B1ft B1 **[7]**

Question Number	Scheme	Marks
3. (a)	$\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx} = 0 \qquad e^{x\sqrt{3}}(\sqrt{3}\sin 3x + 3\cos 3x) = 0$	M1
	$\tan 3x = -\sqrt{3}$	A1
	$3x = \frac{2\pi}{3} \Rightarrow x = \frac{2\pi}{9}$	M1A1
		(6)
(b)	At $x = 0 \qquad \frac{dy}{dx} = 3$	B1
	Equation of normal is $-\frac{1}{3} = \frac{y-0}{x-0}$ or any equivalent $y = -\frac{1}{3}x$	M1A1
		(3)
		(9 marks)

Question Number	**Scheme**	**Marks**
4.	$\left(\frac{dx}{dy}\right) = 2\sec^2\left(y + \frac{\pi}{12}\right)$	M1, A1
	substitute $y = \frac{\pi}{4}$ into their $\frac{dx}{dy} = 2\sec^2\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = 8$	M1, A1
	When $y = \frac{\pi}{4}$. $x = 2\sqrt{3}$ awrt 3.46	B1
	$\left(y - \frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$	M1
	$\left(y - \frac{\pi}{4}\right) = -8\left(x - 2\sqrt{3}\right)$ oe	A1
		(7 marks)

Question Number	Scheme		Marks
5.			
(a)	$\theta = 20 + Ae^{-kt}$ (eqn *)		
	$\{t = 0, \theta = 90 \Rightarrow\}$ $90 = 20 + Ae^{-k(0)}$	Substitutes $t = 0$ and $\theta = 90$ into eqn *	M1
	$90 = 20 + A \Rightarrow \underline{A = 70}$	$\underline{A = 70}$	A1
			(2)
(b)	$\theta = 20 + 70e^{-kt}$		
	$\{t = 5, \theta = 55 \Rightarrow\}$ $55 = 20 + 70e^{-k(5)}$ $\frac{35}{70} = e^{-5k}$	Substitutes $t = 5$ and $\theta = 55$ into eqn * and rearranges eqn * to make $e^{\pm 5k}$ the subject.	M1
	$\ln\left(\frac{35}{70}\right) = -5k$	Takes 'lns' and proceeds to make '$\pm 5k$' the subject.	dM1
	$-5k = \ln\left(\frac{1}{2}\right)$		
	$-5k = \ln 1 - \ln 2 \Rightarrow -5k = -\ln 2 \Rightarrow \underline{k = \frac{1}{5}\ln 2}$	Convincing proof that $k = \frac{1}{5}\ln 2$	A1 *
			(3)
(c)	$\theta = 20 + 70e^{-\frac{1}{5}t\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{1}{5}\ln 2.(70)e^{-\frac{1}{5}t\ln 2}$	$\pm\alpha e^{-kt}$ where $k = \frac{1}{5}\ln 2$ $-14\ln 2 e^{-\frac{1}{5}t\ln 2}$	M1 A1 oe
	When $t = 10$, $\frac{d\theta}{dt} = -14\ln 2 e^{-2\ln 2}$		
	$\frac{d\theta}{dt} = -\frac{7}{2}\ln 2 = -2.426015132...$		
	Rate of decrease of $\theta = 2.426\ {}^{\circ}C/\text{min}$ (3 dp.)	awrt ± 2.426	A1
			(3) **[8]**

Question Number	Scheme	Marks
6(a)	$\ln(4-2x)(9-3x), = \ln(x+1)^2$	M1, M1
	So $36-30x+6x^2 = x^2+2x+1$ and $5x^2-32x+35=0$	A1
	Solve $5x^2-32x+35=0$ to give $x=\frac{7}{5}$ oe (Ignore the solution $x=5$)	M1 A1 (5)
(b)	Take $\log_e$'s to give $\ln 2^x + \ln e^{3x+1} = \ln 10$	M1
	$x\ln 2 + (3x+1)\ln e = \ln 10$	M1
	$x(\ln 2 + 3\ln e) = \ln 10 - \ln e \Rightarrow x = ..$	dM1
	and uses $\ln e = 1$	M1
	$x = \frac{-1+\ln 10}{3+\ln 2}$	A1 (5)
		[10]

Question Number	*Scheme*		Marks
7. (*a*)	$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$ M1 Use of common denominator to obtain single fraction		M1
	$=\frac{1}{\cos\theta\sin\theta}$ M1 Use of appropriate trig identity (in this case $\sin^2\theta+\cos^2\theta=1$)		M1
	$=\frac{1}{\frac{1}{2}\sin 2\theta}$	Use of $\sin 2\theta = 2\sin\theta\cos\theta$	M1
	$= 2\text{cosec}2\theta$ (✱)		A1 cso (4)
(*b*)		Shape (May be translated but need to see 4"sections") T.P.s at $y = \pm 2$, asymptotic at correct x-values (dotted lines not required)	B1 B1 dep. (2)
(*c*)	$2\text{cosec}2\theta = 3$ $\sin 2\theta = \frac{2}{3}$ Allow $\frac{2}{\sin 2\theta} = 3$ [M1 for equation in $\sin 2\theta$]		M1, A1
	(2θ) = [41.810…°, 138.189…° ; 401.810…°, 498.189…°] 1st M1 for $\alpha, 180-\alpha$; 2nd M1 adding 360° to at least one of values		M1; M1
	θ = 20.9°, 69.1°, 200.9°, 249.1° (1 d.p.) awrt		A1,A1 (6)

Question Number	Scheme	Marks
Q8	$\text{cosec}^2 2x - \cot 2x = 1, \quad (\text{eqn } *) \qquad 0 \le x \le 180^\circ$	
	Using $\text{cosec}^2 2x = 1 + \cot^2 2x$ gives $1 + \cot^2 2x - \cot 2x = 1$	M1
	$\underline{\cot^2 2x - \cot 2x} = 0 \quad \text{or} \quad \cot^2 2x = \cot 2x$	A1
	$\cot 2x(\cot 2x - 1) = 0 \quad \text{or} \quad \cot 2x = 1$	dM1
	$\cot 2x = 0 \quad \text{or} \quad \cot 2x = 1$	A1
	$\cot 2x = 0 \Rightarrow (\tan 2x \to \infty) \Rightarrow 2x = 90, 270$	
	$\Rightarrow x = 45, 135$	
		M1
	$\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$	
	$\Rightarrow x = 22.5, 112.5$	
	Overall, $x = \{22.5, 45, 112.5, 135\}$	A1 B1
		[7]

Question Number	**Scheme**	**Marks**
9(a)	$0 \le f(x) \le 10$	B1 (1)
(b)	ff(0) = f(5), = 3	B1, B1 (2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y - 4 = xy + 3x$	M1
	$\Rightarrow 5y - 4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$g^{-1}(x) = \dfrac{5x-4}{3+x}$	A1 (3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4$ oe	M1 A1
	$f(x) = 4 \Rightarrow x = 6$	B1
	$f(x) = 4 \Rightarrow 5 - 2.5x = 4 \Rightarrow x = 0.4$ oe	M1 A1 (5)
		[11]

Statistics for C3 Practice Paper G2

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	4	4	68	2.71	3.64	3.19	2.81	2.48	2.12	1.81	1.31
2	7	7	68	4.77	6.50	5.83	5.08	4.34	3.56	2.77	1.69
3	9		62	5.56	8.61	7.40	5.94	4.41	2.88	1.60	0.57
4	7		58	4.04	6.80	5.90	4.80	3.63	2.54	1.69	0.45
5	8		59	4.68	7.37	6.26	5.19	4.42	3.62	2.74	1.92
6	10	10	55	5.49	9.39	7.46	5.66	4.23	3.08	2.10	1.07
7	12		69	8.24		10.40	8.65	7.42	5.93	4.39	2.51
8	7		43	3.00		5.23	3.39	2.37	1.46	0.73	0.37
9	11	4	45	4.99	8.50	6.41	4.95	3.90	3.02	2.11	1.19
	75		**58**	**43.48**		**58.08**	**46.47**	**37.20**	**28.21**	**19.94**	**11.08**